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Searching for the Origin of CP violation in Cabibbo Suppressed D-meson Decays.

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The recent evidence of relatively large direct CP violation in D^0 decay at LHCb suggests that CP studies in the D system may become an important new avenue for understanding CP just as studies in the B system have proven to be. The current level of CP violation could be consistent with the Standard Model or, perhaps, contain evidence of new physics. A clean Standard Model prediction of the CP violation in these decays would, of course, be important in understanding these results but hadronic uncertainties makes such a prediction difficult. In this paper, we make several suggestions to try seek the role of new physics. We propose that the hadronic enhancement needed to attribute the observed CP violation in D to two pseudoscalar modes may not operate for inclusive final states where it is likely that we will see asymmetries at the quark level expectation provided the source is the Standard Model. A simple way to implement this is to search for CP asymmetries in final states containing K and $\bar{K}$ but where the sum of their energies is less than the energy of the parent D. This is meant to ensure that the event belongs to an inclusive and not an exclusive sample. We also propose that CP asymmetries may be enhanced in modes where the tree is color suppressed. In particular, the final state $\rho^0\rho^0$ is of special interest because it consists of charged pions only and, in addition, it can have C-even P-odd triple product correlations; similarly $D_s \to \rho^0 K^+$ and $\rho^0 K^{*+}$ also appear interesting. We also emphasize the use of CPT constraints leading to interesting correlations. We then consider how isospin symmetry can provide observables which are sensitive to certain classes of new physics and are small in the Standard Model. In particular, we discuss using isospin analysis in the decays $D \to \pi\pi$, $\rho\pi$ and $\rho\rho$ as well as in $D_s \to K^*\pi$. We also consider how such analysis may eventually be supplemented by information about the weak phases in D^0 decay. In order to obtain this information experimentally, we consider various methods for preparing an initial state which is a quantum mechanical mixture of D^0 and $\overline{D}^0$. This may be done through the use of natural $D^0/\overline{D}^0$ oscillations; observing D^0 mesons which arise from B_d or B_s mesons which themselves are oscillating or from quantum correlations in D^0 pairs which arise from either ψ'' decay or B-meson decay. Observing CP violation in the magnitudes of decay amplitudes should be within the capability of experiments in the near future, however, obtaining the weak phases through the methods we discuss will likely require future generations of machines due to the large statistics that are likely to be needed.

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I. INTRODUCTION

Recent results [1] from the LHCb provide evidence for CP violation in D-meson decays, in particular, $A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = -0.82 \pm 0.21(\text{stat}) \pm 0.11(\text{syst})\%$ giving a 3.5 sigma signal of CP violation. CDF has reported [2] the modes separately obtaining $A_{CP}(K^+K^-) = -0.24 \pm 0.22 \pm 0.09\%$ and $A_{CP}(\pi^+\pi^-) = +0.22 \pm 0.24 \pm 0.11\%$. CDF has also directly measured the difference previously measured by LHCb and obtained[3] $A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = -0.62 \pm 0.21(\text{stat}) \pm 0.10(\text{syst})\%$. A similar result of $A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = -0.87 \pm 0.41(\text{stat}) \pm 0.06(\text{syst})\%$ was also recently reported by BELLE at ICHEP2012 [4]. Belle [4] also gave $A_{CP}(K^+K^-) = -0.32 \pm 0.21 \pm 0.09\%$ and $A_{CP}(\pi^+\pi^-) = +0.55 \pm 0.36 \pm 0.09\%$. In the LHCb result, the cancellation of experimental uncertainties between the two modes plays an important role in the extraction of a significant signal for the difference in the two asymmetries. These measurements dominate the world average for the difference given by the HFAG group[5]: $A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-) = -0.678 \pm 0.147\%$. The Belle results are particularly significant for individual modes, because the leptonic environment allows better detection of these two final states, so super KEK [6, 7] and the Super-B Factory [8] should be able to produce more precise results in the future especially for individual modes.

The LHCb result for the difference in the asymmetries appears to be large compared to the Standard Model(SM) [9] based on early expectations as we will discuss below. The weak phase arises in the SM from the CKM matrix. The relevant combination of CKM elements which gives the weak phase between the tree and penguin graphs is

$$|\theta_W| \approx 5.6 \times 10^{-4} \tag{1}$$

where we have expanded this in terms of the the Wolfenstein parametrization[10]. As discussed in Section II we expect the CP asymmetry on the *quark level* to be roughly of this size. For specific final states, hadronic effects will alter this expectation appreciably especially since charm quark is so light. Thus the central value for $A_{CP}(K^+K^-) - A_{CP}(\pi^+\pi^-)$ may not be inconsistent with the SM [11–15] but this is far from a proof that the SM is fully adequate and therefore the role of new physics cannot yet be ruled out [11, 12, 16, 17]. There is, in fact, the intriguing possibility that various models for new physics (NP) are also able to contribute significantly to direct CP violation in D-decays [11, 17–28]. For this reason and others, it is important to devise observables which can distinguish between SM and NP origins for this CP violation.

Broadly speaking, for singly Cabibbo suppressed (SCS) decays it is useful to divide potential models of NP into two categories. The first is "penguin like" where the effective Hamiltonian is strictly $\Delta I = \frac{1}{2}$. Generally this kind of contribution will result only when the NP contributes through a gluonic penguin. The second is "tree like" where the effective Hamiltonian contains a $\Delta I = \frac{3}{2}$ component. This includes models where there are extra massive scalar or vector bosons which enter at tree level as well as photon, Z or W penguin topologies. In principle, electroweak penguins could contribute in this way but in the SM such contributions are negligible.

Depending on whether the new physics is tree like or penguin like, different kinds of studies may help identify the underlying mechanism. In this paper we consider strategies which would be helpful in both cases.

In section II the requirements of CPT symmetry motivates us to devise a general test for SM versus NP. If the large observed CP asymmetry is due to SM alone, then we would expect CP violation in a more inclusive final state to converge to the SM quark level expectation given in eqn. 1. Using the number of kaons as a surrogate for the number of s-quarks, we suggest that CP violation in inclusive $K\overline{K} + X$ final states would provide a good test of this idea. We also suggest that hadronic matrix element enhancements mostly occur in only exclusive two-body modes (especially pseudoscalars). With that in mind a simple method is suggested to experimentally identify inclusive events.

In section III we focus on finding additional two body decay modes which are likely to also show large CP asymmetries in the case of penguin-like NP. The key point here is to go after color - suppressed tree modes. In selecting potentially useful modes, we also consider which final states are more easily detected because they appear in the final state as charged particles only. Using these criteria modes of particular interest include $D^0 \to \rho^0\rho^0$, $D_s \to \rho^0 K^{(*)+}$ and $D^0 \to K^{(*)0}\overline{K}^{(*)0}$. In addition, the final states $\eta\eta$, $\eta\eta'$ and $\eta\phi$ may be interesting because the "s-quark rich" nature of the final states even though they contain neutrals in the final state; these could be of special interest to upcoming SuperB Factories. We briefly discuss radiative final states which are considered in [29]. These modes may show CP violation in a large class of penguin NP models, however through CPT arguments we show that A_{CP} is likely to be suppressed.

In all of the above, CP violation is observed through A_{CP}, a difference in decay rate between D and $\overline{D}$ to a given final state. For this form of CP violation a strong phase is also required. It would be very useful to also be able to measure the weak phase directly, independently of the strong phase. In section IV we propose various methods to measure this phase. Three methods are considered: (1) using D^0 oscillation just as oscillation in B mesons are used to measure the weak phase in $B^0 \to \psi K_s$; (2) using oscillation in B-mesons where $B \to \overline{D}^0\rho^0$ (or similar final states) and (3) using correlations in $D^0\overline{D}^0$ pairs. If we assume that the phase between D^0 and $\overline{D}^0$ decay to a given final state is of the same magnitude as the observed value of A_{CP} in $D \to \pi\pi$ and $D \to KK$ then the statistics required is $\sim 10^{11}$ mesons for the methods using oscillation in D-mesons, oscillation in D-mesons and correlations in D-pairs originating from B-meson decay. In the case where the D-pairs arise in a ψ'' factory, then $\sim 10^9$ D-mesons are required. More realistically, weak phases an order of magnitude larger than the currently observed A_{CP} may be observable in the foreseeable future. This would indicate a situation where the strong phase is small and yet there is a large NP weak phase.

In section V we consider tests for NP based on isospin which would apply to tree-like NP models. In some cases isospin can be used to isolate CP violation in the $\Delta I = \frac{3}{2}$ channel. Since the SM predicts no CP violation in this channel, such a signal would indicate the presence of NP.

Tests of this form where the magnitude of a D amplitude is compared to that of a $\overline{D}$ decay amplitude are proposed for $D \to \pi\pi$, $D \to \rho\pi$, $D_s \to \pi K^*$ and $D \to \rho\rho$. In the case of $D \to \rho\pi$ there are two separate amplitudes which can be used, one derived only from the $D^0 \to \pi^+\pi^-\pi^0$ overall reaction and one which also includes input from $D^+ \to \rho^+\pi^0$; $\rho^0\pi^+$. In the case of $D \to \rho\rho$ each polarization can provide a separate test.

For the final states $\pi\pi$, $\rho\rho$ and $\rho\pi$, we can also combine this analysis with weak phase determination in section IV then additional tests are possible where the phase of a $\Delta I = \frac{3}{2}$ amplitude is compared to its conjugate. Again such a phase would be indicative of tree-like NP.

In Section VI we discuss the statistical requirements for testing the SM, particularly for the determination of weak phases. In Section VII we give our summary and conclusion.

II. CPT AND FLAVOR SYMMETRY CONSIDERATIONS

A. Rough Estimate for Quark Level Expectations

The effective Hamiltonian for SCS charm decay can be written:

$$H_{eff} = \frac{G_F}{\sqrt{2}}\left\{\frac{1}{2}(\lambda_s-\lambda_d)\sum_{i=1,2}C_i(Q_i^s-Q_i^d)-\lambda_b\left(\sum_{i=1,2}\frac{1}{2}C_i(Q_i^s+Q_i^d)+\sum_{i=3,6}C_iQ_i\right)\right\}+h.c. \tag{2}$$

where $\lambda_q = V_{cq}V_{uq}^*$ (note that $\lambda_d+\lambda_s+\lambda_b=0$ by CKM unitarity). The operators are

$$\begin{array}{ll}
Q_1^q = (\overline{q}u)_{V-A}(\overline{c}q)_{V-A} & Q_2^q = (\overline{q}_\alpha u_\beta)_{V-A}(\overline{c}_\beta q_\alpha)_{V-A} \\
Q_3 = \sum_{q=u,d,s}(\overline{c}u)_{V-A}(\overline{q}q)_{V-A} & Q_4 = \sum_{q=u,d,s}(\overline{c}_\alpha u_\beta)_{V-A}(\overline{q}_\beta q_\alpha)_{V-A} \\
Q_5 = \sum_{q=u,d,s}(\overline{c}u)_{V-A}(\overline{q}q)_{V+A} & Q_6 = \sum_{q=u,d,s}(\overline{c}_\alpha u_\beta)_{V+A}(\overline{q}_\beta q_\alpha)_{V-A}
\end{array} \tag{3}$$

The first term proportional to $\lambda_s-\lambda_d$ is the tree contribution and the term proportional to λ_b is the penguin contribution. If we assume that there is a strong phase difference between the tree and penguin of ϕ_{strong} then we can write the CP asymmetry in the quark level process $c\to d\overline{d}u$ as [31]:

$$\begin{aligned}
\left|A_{CP}(c\to d\overline{d}u)\right| &= \left|Im\left(\frac{2\lambda_b}{\lambda_s-\lambda_d}\right)\right|R\sin\phi_{strong}\approx\left|Im\left(\frac{V_{ub}V_{cb}^*}{V_{us}V_{cs}^*}\right)\right|R\sin\phi_{strong} \\
&\approx A^2\lambda^4\eta R\sin\phi_{strong}\approx 6.4\times10^{-4}\sin\phi_{strong}
\end{aligned} \tag{4}$$

Here R is a number of order 1 which depends on the Wilson coefficients. If we neglect the mass of the s-quark and hadronization effects and use the one loop evolution of the Wilson coefficients given in [32, 33] then numerically $R\approx 1.2$. The resultant asymmetry at the quark level is thus expected to be about the same as given in Eqn. 1 if the strong phase is near maximal. As discussed below CPT implies $\Delta\Gamma(c\to d\overline{d}u)=-\Delta\Gamma(c\to s\overline{s}u)$ where for a given decay $A\to B$, $\Delta\Gamma(A\to B)=\Gamma(A\to B)-\Gamma(\overline{A}\to\overline{B})$

This quark level result need not be the same as the CP asymmetry in any given exclusive hadronic final state. Due to the significant hadronic uncertainties in the formation of specific final states, in order to characterize the CP violation in D-meson decay it is useful to consider symmetries which may be partially respected by strong interactions. The most obvious such symmetry is $SU(3)_{flavor}$ which, unfortunately is badly broken [17]. We will discuss the use of the isospin subgroup of $SU(3)$ in section V. For the current experimental results, a more handy subgroup of $SU(3)$ to consider is U-spin since it directly relates the K^+K^- and $\pi^+\pi^-$ final states. If this symmetry were strictly observed then $Br(D^0\to\pi^+\pi^-)=Br(D^0\to K^+K^-)$ and $A_{CP}(D^0\to\pi^+\pi^-)=-A_{CP}(D^0\to K^+K^-)$. Since $Br(D^0\to\pi^+\pi^-)=(1.397\pm0.026)\times10^{-3}$ while $Br(D^0\to K^+K^-)=(3.94\pm0.07)\times10^{-3}$, more than a factor of 2 discrepancy, it is clear that U-spin is badly broken. Due to the large experimental error in the individual CP asymmetries, no firm conclusion with respect to U-spin applied to CP violation can be drawn though the central values tend to show opposite sign.

B. CPT Relations

Another symmetry which must, of course, be respected, is CPT which implies that the width of the D and $\overline{D}$ mesons must be the same. This means that when summed over all final states of D-meson decay, the partial rate differences must vanish:

$$\sum_X\Delta\Gamma(X)=0 \tag{5}$$

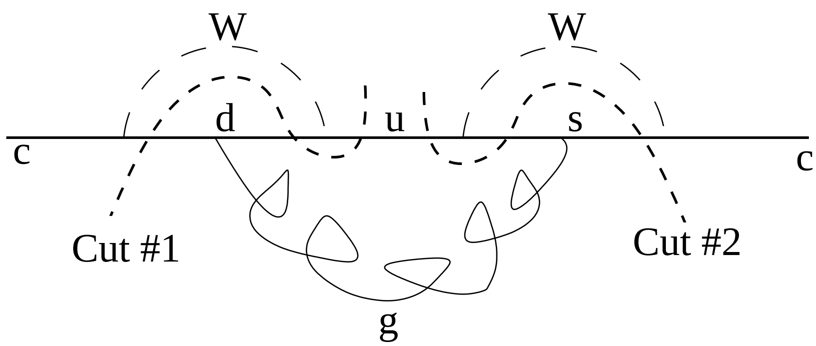

FIG. 1: The unitarity graph showing the CPT identity Eqn. 6 for the quark level SCS charm decay. Cut #1 indicated in the figure shows the case where the decay is $c \to d\overline{d}u$ with a $s\overline{s}u$ intermediate state providing the strong phase. Conversely, cut #2 indicated in the figure shows the case where the decay is $c \to s\overline{s}u$ with a $d\overline{d}u$ intermediate state providing the strong phase. The interfering tree graphs are not shown but are implied

In detail, this means [34, 35] that $\Delta\Gamma$ must be exchanged between the various final states. This exchange is caused by rescattering between at least two final states (say X_1 and X_2) with different strong and weak phases. If X_2 rescattering into X_1 provides a strong phase for X_1 it will give rise to a contribution to $\Delta\Gamma(X_1)$. This partial rate asymmetry will be exactly canceled by the contribution to $\Delta\Gamma(X_2)$ proportional to the strong phase produced when X_1 rescatters into X_2.

At the quark level, the SM maintains CPT in SCS decays by an exchange of $\Delta\Gamma$ between $c \to ud\overline{d}$ and $c \to us\overline{s}$, hence

$$\Delta\Gamma(c \to d\overline{d}u) = -\Delta\Gamma(c \to s\overline{s}u). \tag{6}$$

The "double penguin" unitarity graph in Fig. 1 shows how this compensation arises where the two cuts indicate the two final states. Thus, cut #1 gives a final state with $d\overline{d}u$ where one of the amplitudes has an internal loop with an $s\overline{s}u$ final state. The magnitude of this graph is the same as that given by cut #2 giving a $s\overline{s}u$ final state with an intermediate $d\overline{d}u$ but the sign is opposite due to the internal loop being on the left side in this case.

To draw conclusions concerning specific groups of decay modes, it is useful to break down Eqn. 6 according to quantum numbers conserved by strong interactions, since the exchange of $\Delta\Gamma$ between final states can only occur between states which can rescatter into each other. Such rescattering is via strong interactions so the general statement in Eqn. 5 can be refined to:

$$\Delta\Gamma(P, C, I, G, S) = 0 \tag{7}$$

where P, C, I, G, S are the quantum numbers, parity, charge conjugation, isospin, G-parity and strangeness respectively.

In applying this to SCS modes, where $S = 0$ in the final state, we can further classify final states according to the number, N_K, of kaons and anti-kaons they contain. Notice that in general $N_K \in \{0, 1, 2, 3\}$ because $M_D < 4M_K$ and for S=0 in particular, $N_K \in \{0, 2\}$ therefore

$$\Delta\Gamma(PCIG, S = 0, N_K = 0) = -\Delta\Gamma(PCIG, S = 0, N_K = 2) \tag{8}$$

The left and right sides of these equations should represent $d\overline{d}u$ and $s\overline{s}u$ quark content respectively since it is expected that $c \to ud\overline{d}$ couples dominantly to $N_K = 0$ while $c \to us\overline{s}$ couples dominantly to $N_K = 2$ so $\Delta\Gamma(c \to ud\overline{d}) \sim \Delta\Gamma(N_K = 0)$ and $\Delta\Gamma(c \to us\overline{s}) \sim \Delta\Gamma(N_K = 2)$.

Implementing Eqn. 8 directly for each combination of quantum numbers is difficult since many of the D decay modes contain multiple charged pions and so the quantum numbers may be difficult to determine on an event by event basis.

What may be a more practical an experimental test of CP violation is to look for CP violation in the inclusive case summed over P, C, I, G. In this case CPT implies:

$$\hat{\Delta}\Gamma(S = 0, N_K = 0) + \Delta\Gamma(\pi + \pi) = -\hat{\Delta}\Gamma(S = 0, N_K = 2) - \Delta\Gamma(K\ K) \tag{9}$$

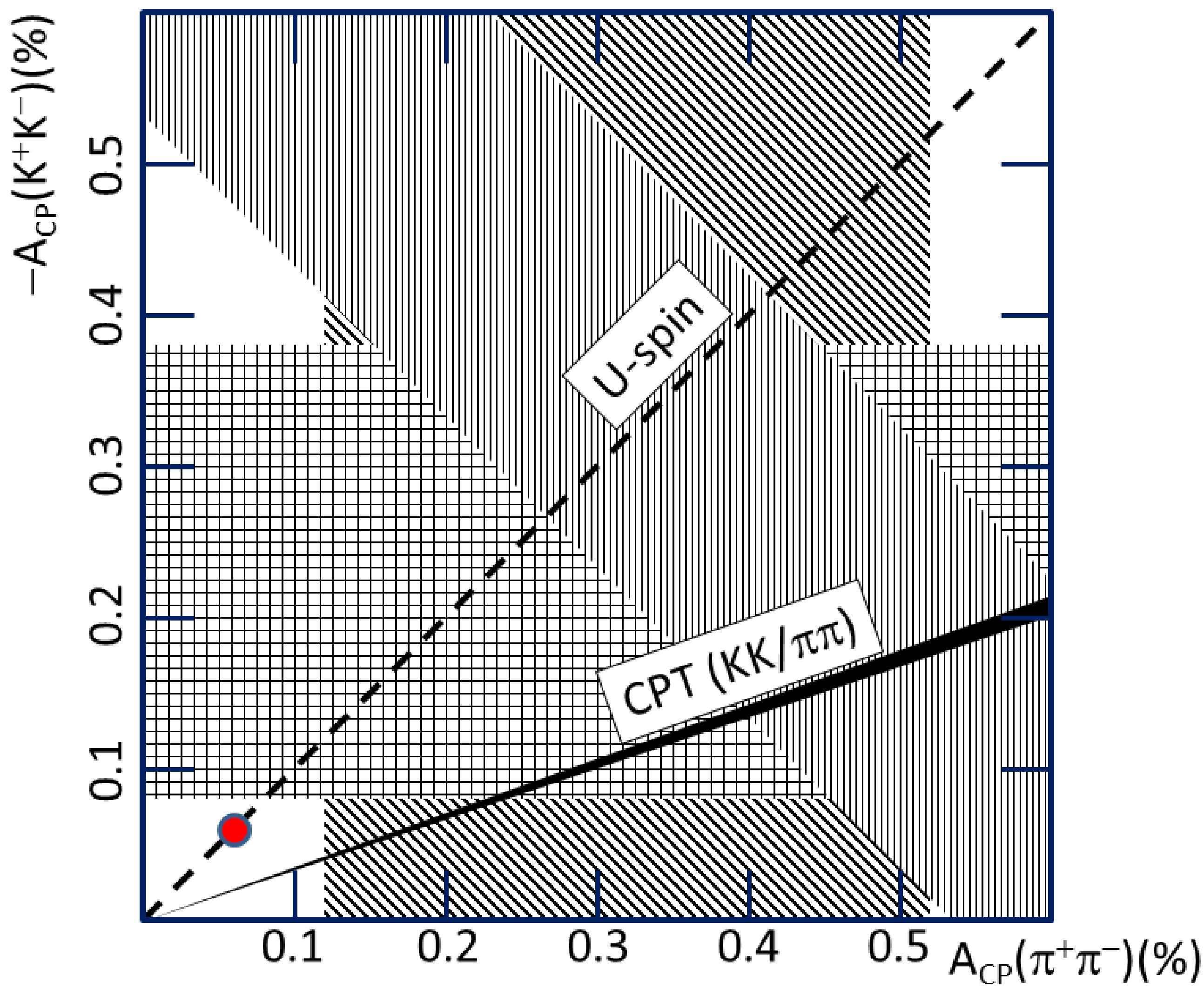

FIG. 2: The current experimental results for $A_{CP}(\pi^+\pi^-)$ and $A_{CP}(K^+K^-)$. The vertically hatched band shows the 1-σ region for the measurement of $A_{CP}(\pi^+\pi^-) - A_{CP}(K^+K^-)$; the diagonally hatched band shows the 1-σ region for the measurement of $A_{CP}(\pi^+\pi^-)$; the square hatched band shows the 1-σ region for the measurement of $A_{CP}(K^+K^-)$. The dashed line indicates the U-spin prediction that $A_{CP}(\pi^+\pi^-) + A_{CP}(K^+K^-) = 0$. The black wedge indicates the result where CPT is maintained between K^+K^- and $\pi^+\pi^-$, i.e. $\Delta\Gamma(K^+K^-) + \Delta\Gamma(\pi^+\pi^-) = 0$. The dot towards the lower right is the quark level expectation from Eqn. 4 with maximal strong phase if you make the naive assumption that $A_{CP}(K^+K^-) = A_{CP}(s\bar{s}u)$ and $A_{CP}(\pi^+\pi^-) = A_{CP}(d\bar{d}u)$

where $\hat{\Delta}$ means that two body pseudoscalars are not included as we explain more below. CP asymmetry in both $\hat{\Delta}\Gamma(S = 0, N_K = 0)$ and $\hat{\Delta}\Gamma(S = 0, N_K = 2)$ should approximate the quark level CP asymmetry in $d\bar{d}u$ and $s\bar{s}u$ respectively.

If CP violation in the PP final states is due to the SM then there must be some hadronic enhancement for those exclusive final states such as $\pi^+\pi^-$, K^+K^-, which we would not expect to be present in the inclusive Eqn. 9. Recall also that for exclusive two pseudosaclar modes, in particular, there are well known reasons to expect large QCD corrections, *e. g.* chiral enhancements. It is quite unlikely that inclusive modes will receive such largish enhancements.

Thus, the inclusive CP asymmetry should be smaller, $\sim 6\times 10^{-4}$. On the other hand, if the largeness of the CP asymmetry in the PP final states is due to NP, then one would expect the inclusive asymmetry to be roughly of the same order as the exclusive. The larger statistics of the inclusive state may provide more accurate results for this channel leading to an important indication of the nature of the observed CP violation.

In practice, observing a quantity like $\hat{\Delta}\Gamma(S=0,N_K=2)$ is subject to the problem that it is not possible to catch every final state. Thus it is useful to rephrase the relation $\hat{\Delta}\Gamma(S=0,N_K=2)\sim\hat{\Delta}\Gamma(c\to s\bar{s}u)$ as

$$lim_{\chi\to I}\hat{\Delta}\Gamma(S=0,N_K=2;\chi)\sim\hat{\Delta}\Gamma(c\to s\bar{s}u) \tag{10}$$

where χ is some CP invariant acceptance cut on the final states and I represents the cut where all events are accepted. In any case $\hat{\Delta}\Gamma(S=0,N_K=2;\chi)$ is a CP violating quantity.

Actually, given that in the sample of inclusive ($N_K=2$) final states we do not want to include the exclusive $K\overline{K}$ mode, a simple working definition of inclusive is all those final states in which the sum of K and $\overline{K}$ energies is less than the energy of the parent D.

Eqn. 8 can be broken down further with approximate symmetries allowing us to gain some understanding of the pattern of CP violation in exclusive decay modes. For instance, if U-spin were a good symmetry then Eqn. 8 could be broken down into groups of final states related by this symmetry. In particular it would follow that $\Delta\Gamma(\pi^+\pi^-)=-\Delta\Gamma(K^+K^-)$. Of course U-spin is broken but in [11] figure 2 they fit the experimental data including U-spin breaking allowed within the SM. This fit favors a solution where $|A_{CP}(\pi^+\pi^-)/A_{CP}(K^+K^-)|>1$ although the statistics are not yet good enough to draw any firm conclusion. Analogously, in [15] (see eq 34), the above ratio of asymmetries is predicted to be ≈ 1.8. This suggests a pattern where the partial rate asymmetry exchange is mostly between these two final states, in which case we would expect $A_{CP}(\pi^+\pi^-)/A_{CP}(K^+K^-)\approx -Br(K^+K^-)/BR(\pi^+\pi^-)\approx -2.82\pm 0.14$.

In Figure 2 the current weighted average results for $A_{CP}(\pi^+\pi^-)$ and $A_{CP}(K^+K^-)$ are shown as 1σ bands on a $A_{CP}(\pi^+\pi^-)$ versus $A_{CP}(K^+K^-)$ plot (indicated by the diagonally hatched and square hatched regions respectively) as well as the world average for the $A_{CP}(K^+K^-)-A_{CP}(\pi^+\pi^-)$ result (vertically hatched band). The dashed line indicates the U-spin result $A_{CP}(K^+K^-)=-A_{CP}(\pi^+\pi^-)$ while the black wedge indicates the result where we assume that CPT is maintained within PP final states by exchange between K^+K^- and $\pi^+\pi^-$, i.e. that $\Delta\Gamma(K^+K^-)+\Delta\Gamma(\pi^+\pi^-)=0$. For comparison the circle in the lower left corner indicates the naive expectation where we assume that the meson asymmetry is the same as the quark level asymmetry. In particular we suppose here that the quark level asymmetry is given by Eqn. 4 with maximal strong phase and that $A_{CP}(K^+K^-)=A_{CP}(s\bar{s}u)$ and $A_{CP}(\pi^+\pi^-)=A_{CP}(d\bar{d}u)$.

In order to facilitate distinguishing SM from NP contributions, in Section (V) we will discuss relations which rely on isospin only which, unlike the more general $SU(3)$, should be good to the level of a few percent.

III. CANDIDATES FOR ENHANCED CP VIOLATION FOR PENGUIN-LIKE NEW PHYSICS

Suppose that CP violation is the result of a large amplitude A interfering with a smaller amplitude a. If we normalize the amplitudes in units of square root of branching ratio, then $Br(D\to f)\approx |A|^2$ while $A_{CP}(f)\propto a/A$. If we want to observe the CP violation with a significance of N_σ, the number of mesons required is $N=N_\sigma^2/(BrA_{CP}^2)$. In terms of the amplitudes then,

$$N=N_\sigma^2/(BrA_{CP}^2)\propto\frac{N_\sigma^2}{|A|^2|a/A|^2}\propto\frac{N_\sigma^2}{|a|^2} \tag{11}$$

So that generally N depends on a but is independent of A but a smaller value of A does enhance A_{CP}; N is not affected because this is at the expense of the branching ratio. Going to a mode which has smaller branching ratio with higher asymmetry has the advantage of reducing the effects of systematic errors and other errors which are not statistical in nature, *all other things being equal*.

If we assume that the observed CP violation in $D^0\to\pi^+\pi^-$, K^+K^- is due to penguin like NP it may be that larger signals of CP asymmetries will be present in similar decays where the SM tree contribution is suppressed. Following this rationale, in this section we focus on the cases where the SM tree is color suppressed.

Color suppression in two body final states is a pattern which is often born out in B-meson decays. For example in decays to charm mesons the color allowed $B^0\to\pi^+D^-$ has a branching ratio of $(2.6\pm0.13)\times10^{-3}$ while the analogous color suppressed mode $B^0\to\pi^0\overline{D}^0$ has a branching ratio an order of magnitude smaller of $(2.61\pm0.24)\times10^{-4}$. A similar pattern obtains for related decays. In contrast, in the case of B-meson decay to two light pseudoscalar mesons, color suppression fails. Thus $Br(B^0\to\pi^+\pi^-)=(5.13\pm0.24)\times10^{-6}$ while $Br(B^0\to\pi^0\pi^0)=(1.62\pm0.31)\times10^{-6}$

but with PV and PP final states, for instance $Br(B^0 \to \pi^+\rho^- + \pi^-\rho^+) = (2.3 \pm 0.23) \times 10^{-5}$ versus $Br(B^0 \to \pi^0\rho^0) = (2.0 \pm 0.5) \times 10^{-6}$, likewise $Br(B^0 \to \rho^+\rho^-) = (2.42 \pm 0.31) \times 10^{-5}$ versus $Br(B^0 \to \rho^0\rho^0) = (7.3 \pm 2.6) \times 10^{-7}$; color suppression of these modes seems to hold.

It is to be expected that color suppression is less effective in D decays because of greater non-perturbative effects and increased meson rescattering at the charm mass scale. Indeed this appears to be the case, for example $Br(D^0 \to K^-\pi^+) = 3.89\%$ while $Br(D^0 \to \bar{K}^0\pi^0) = 2.44\%$ showing no color supression, likewise $Br(D^0 \to K^{*-}\pi^+) = 1.73\%$ while $Br(D^0 \to \overline{K}^{*0}\pi^0) = 2.28\%$ again showing no color suppression. Conversely the $K\rho$ channel does seem to show the effect since $Br(D^0 \to K^-\rho^+) = 10.8\%$ while $Br(D^0 \to \overline{K}^0\rho^0) = 1.32\%$.

In spite of the unreliable evidence that color suppression is universally operative in D-meson decays, used with care and caution, it may provide us with a guide in searching for other decay modes for future searches for enhanced CP violation. In particular, modes where the SM amplitude tends to be suppressed and a possible NP penguin amplitude may be enhanced may be of particular interest in NP searches. In table I we list all the two body Cabibbo suppressed decay modes of D-mesons to ground state mesons. Here we use the notation $\pi^{(*)\pm}$ to indicate either $\pi^\pm$ or $\rho^\pm$ and $\pi^{(*)0}$ to indicate either π^0, ρ^0 or ω^0. Likewise we use $\phi^{(*)}$ to indicate either ϕ or $\eta^{(\prime)}$ (or at least the $s\overline{s}$ component of the latter).

Decays of the form $D_s \to \pi^{(*)0}K^{(*)+}$, $D^+ \to \pi^{(*)+}\phi^{(*)}$, $D^0 \to K^{(*)0}\overline{K}^{(*)0}$ and $D^0 \to \pi^{(*)0}\phi^{(*)}$, have the tree color suppressed in this way. Also modes of the form $D^0 \to K^{(*)0}\overline{K}^{(*)0}$ have an additional suppression of the tree contribution due to the fact that the $d\overline{d}s\overline{s}$ final state quark content is not the same as produced by the tree graph. This is born out by the smallness of the branching ratios of $D^0 \to K_sK_s$ and $D^0 \to K^{*0}\overline{K}^{*0}$. Therefore, most promising from this list is $D_s \to \pi^{(*)0}K^{(*)+}$.

As an example of how color suppression can work in the realm of Cabibbo Allowed D-meson decays, consider such decays to two vector final states. The decay $D^0 \to K^{*+}\rho^-$ has no suppression and its branching ratio is $10.8 \pm 0.7\%$. The related color suppressed decays $D^0 \to \overline{K}^{*0}\rho^0$ and $D^0 \to \overline{K}^{*0}\omega$ have branching ratios $1.58 \pm 0.35\%$ and $1.1 \pm 0.5\%$ respectively.

In the table, we also enumerate the cases where the mode cascades down to a final state which contains all charged particles (i.e. $\pi^\pm$ and $K^\pm$), and give their branching ratios from [36], where known. Final states with all charged states will generally be easier to detect, particularly at the LHCb. Based on these criteria, $D^0 \to \rho^0\rho^0$, $D_s \to \rho^0K^+$ and $D_s \to \rho^0K^{*+}$ are perhaps the most favorable channels to find CP violation due to penguin like new physics.

From this point of view, the cases of $D^0 \to \rho^0\rho^0$ is perhaps of particular interest to search for enhanced CP violation due to NP. An additional feature of this VV final state is the spin degree of freedom: there are three polarization states: transverse parallel ($A_\parallel$), transverse perpendicular ($A_\perp$) and longitudinal (A_ℓ) . Each amplitude could have different CP violation. Already existing measurements [36] of the polarization fractions show that the longitudinal mode dominates with a fraction of 67% longitudinal. Further measurements of the angular distribution using the methods of [37] will allow the extraction of the phases between the polarization amplitudes.

As discussed in [38, 39] and in [40] for the analogous B decays, a qualitatively different feature of VV final states is that there can be P-odd triple product observables. Such observables can lead to either CP-odd or CP-even correlations depending on the combination of D and $\overline{D}$ decays. If the C-even combination is formed (adding the triple product of D and $\overline{D}$) then the combination is CP-odd conversely the C-odd combination is CP-even. CPT then requires C-odd, CP violating amplitudes to be real whereas C-odd, CP violating amplitudes need a rescattering strong phase.

CP-odd observables of this form that are C-even can be formed from untagged samples of D^0 mesons. This is an advantage for e^+e^- B-factories where the initial state is self conjugate so the D^0 samples obtained at such machines could be used directly (except for the small asymmetry in the D meson production mechanism between inclusive $B \to \overline{D} + X$ versus $\overline{B} \to D + X$ which will have to be determined from separate studies). In the case of LHCb this would not be true since the pp initial state is not self conjugate so tagging will, in any case, be necessary.

In some tree like NP models, even if the SM tree is colored suppressed the NP contribution is not. This will tend to enhance the NP contribution to CP violation. In particular, if a $q\overline{q}$ pair is produced by a color neutral object (e.g. a Z' or higgs like boson) then the effective Hamiltonian will have a different color structure form the SM and so color suppression may not apply. In addition to the two families of modes mentioned above, $D_s \to \pi^{(*)0}K^{(*)+}$, and $D^0 \to \pi^{(*)0}\pi^{(*)0}$, modes of the form $D^+ \to \pi^{(*)+}\phi^{(*)}$ should have enhanced CP asymmetries in this scenario.

Treelike CP violation of this form should contribute to the $\Delta I = \frac{3}{2}$ channel and so the isospin analysis in Section V should reveal this though with some unknown hadronization effects and thus may reveal a contradiction with the SM. In this scenario, the $\pi\pi$, $\rho\pi$ and $\rho\rho$ systems are particularly suited since there are enough charge distributions to allow isospin analysis and the final state with two neutral mesons has a color suppressed tree graph which may enhance NP CP violation. Of these cases, the $\rho^0\rho^0$ state also has the advantage that it leads to an observed final state with all charged mesons.

Another class of final states which may be of interest in some models of new physics are those which are rich in $s\overline{s}$,

in particular if they contain ϕ η and η'. The only three such two body modes which are kinematically allowed are $D^0 \to \phi\eta$, $D^0 \to \eta'\eta$ and $D^0 \to \eta\eta$. Of course none of these modes leads to an all charged final state but the tree graph is color suppressed. There is some additional suppression since the quark content only couples to the $u\overline{u}$ part of the $\eta^{(\prime)}$ wave function which makes up only 20-30% of these mesons.

Another manifestation of penguin-like NP which could lead to CP violating signals are radiative decays. At the quark level such decays would proceed through $c \to u\gamma$ which leads to modes like $D^0 \to \rho^0\gamma$, $D^0 \to \omega\gamma$, $D^+ \to \rho^+\gamma$ and $D_s \to K^{*+}\gamma$. Other radiative D-meson decays which would not be expected to receive large contributions from short distance radiative penguins are $D^0 \to K^{0*}\gamma$ ($Br = (3.28 \pm 0.35) \times 10^{-4}$), $D^0 \to \phi\gamma$ ($Br = (2.70 \pm 0.34) \times 10^{-5}$). In [29] these kind of radiative decays are discussed in the context of new physics. According to their analysis, if the QCD dipole $c \to u$ transition operators:

$$Q_8 = g_s \frac{m_c}{4\pi^2} \overline{u}_L \sigma_{\mu\nu} T^a G_a^{\mu\nu} c_R \quad Q_8' = g_s \frac{m_c}{4\pi^2} \overline{u}_R \sigma_{\mu\nu} T^a G_a^{\mu\nu} c_L \tag{12}$$

then operator evolution from the NP scale to the charm scale would lead to comparable coefficients for the electromagnetic dipole $c \to u$ transition operators:

$$Q_7 = eQ_u \frac{m_c}{4\pi^2} \overline{u}_L \sigma_{\mu\nu} F^{\mu\nu} c_R \quad Q_7' = eQ_u \frac{m_c}{4\pi^2} \overline{u}_R \sigma_{\mu\nu} F^{\mu\nu} c_L \tag{13}$$

Even if the coefficient of $Q_7^{(\prime)}$ were much smaller than Q_8 at the high NP scale, the coefficients may be comparable at the charm scale. Assuming that the observed LHCb result is due to the effects of $Q_8^{(\prime)}$ they [29] suggest that the induced coefficient of $Q_7^{(\prime)}$ would generate A_{CP} in $D \to \rho\gamma$ of $O(10\%)$ provided the strong phases involved were maximal (See however[30]). Using vector dominance one can estimate that $Br(D \to \rho\gamma) \approx 10^{-5}$ perhaps making this mode a good test for new physics.

It is, however, unlikely that the strong phase will be $O(1)$ because the kinematics of the rescattering forces there to be an α_s correction to satisfy CPT. To see this consider the unitarity diagrams in Figure 3 for the quark level process $c \to u\gamma$. Diagram 1 in this figure shows the interference of a NP penguin with the SM penguin (having an internal s quark) at lowest order. Note that while cut #1 corresponds to the $u\gamma$ final state, cut #2 does not correspond to an on shell state since the $s\overline{s}$ pair must rescatter into a single photon. There is therefore no strong phase and so there cannot be a CP asymmetry from this diagram. Diagram 2 shows an order α_s correction to diagram where there can be a strong phase since now cut #2 corresponds to an on shell state. This diagram therefore can give rise to a CP asymmetry but that asymmetry will be suppressed by α_s since there is an extra loop. Model calculations carried out in[30] appear to bear out this situation.

IV. WEAK PHASE DETERMINATION BY INITIAL STATE MIXING

In this section we will consider CP violation which arises in the phase between D^0 and $\overline{D}^0$ decaying to a common final state f. Using the interference between the two amplitudes, the weak phase can be directly measured without relying on the the existence of a strong rescattering phase. Thus it is useful to find new examples of CP violation as well as elucidating the source of CP violation in D-meson decays.

In all cases these kinds of measurements are difficult and most likely cannot be carried out in the near future if the weak phase is of the same order of magnitude of the the currently observed CP asymmetry in PP final states. It is possible that the small direct CP asymmetry seen in $D^0 \to PP$ results from a large weak phase in combination with a small (i.e. $< O(10\%)$) strong phase. If this proves to be the case then a weak phase (O(10%)) may be measured by the various methods considered here. A phase of this magnitude in any mode would be hard to explain in the SM and so would be an indication of NP.

If the CP violation in D-mesons is presumed to be from the SM source, the weak phase measurement can also tell us the magnitude of the penguin contribution. To see this, recall that the SM decay amplitude receives a dominant contribution from the tree which has no weak phase and a penguin contribution with weak phase γ. We can therefore write the amplitude for any SCS decay and its conjugate as:

$$\begin{aligned} A &= T + Pe^{+i\gamma} \\ \overline{A} &= T + Pe^{-i\gamma} \end{aligned} \tag{14}$$

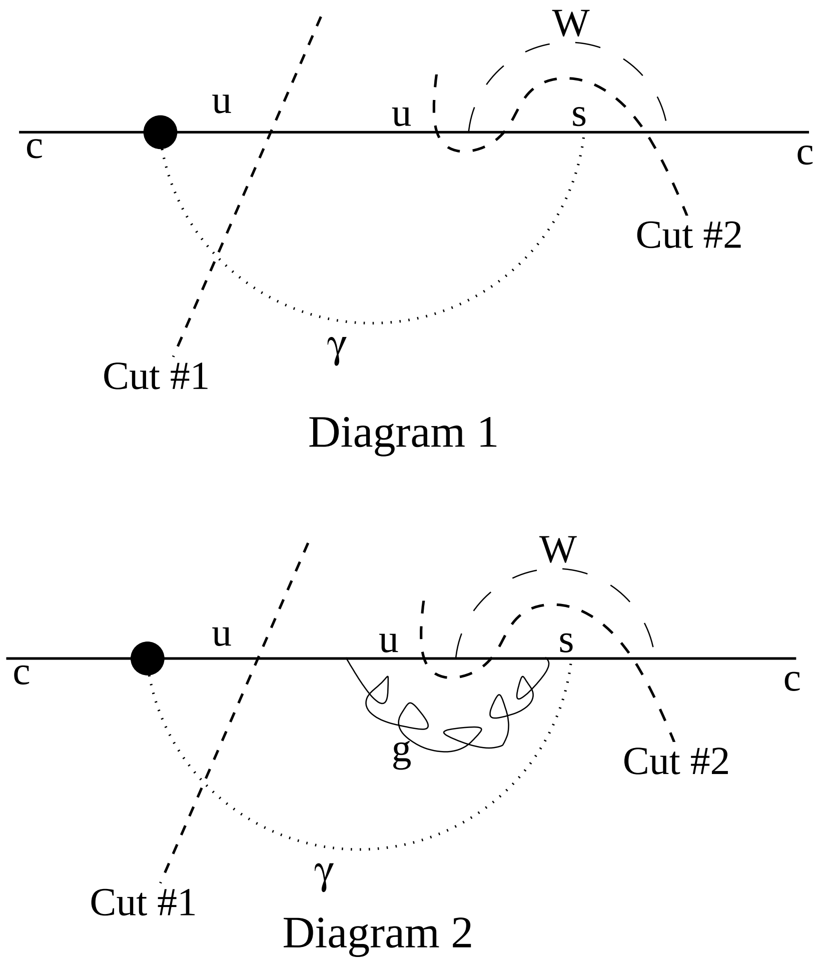

FIG. 3: This unitarity graph illustrates CPT conservation for the quark level process $c \to u\gamma$ due to NP. Diagram 1 shows the lowest order interference between NP and SM where cut #1 is for the $c\gamma$ final state and cut #2 is for a $s\bar{s}u$ final state. Cut #2 cannot be on shell. Diagram 2 shows an example of an order α_s correction to diagram 1 where in contrast cut #2 can be on shell.

where T and P are generally complex numbers because they contain strong phases.

Thus if we know $|A|$ and $|\overline{A}|$ from measurement of D^0 and $\overline{D}^0$ decays to the final state as well as the phase $\theta = arg(\overline{A}A^*)$ then

$$\left||A| - \left|\overline{A}\right| e^{i\theta}\right| = 2|P|\sin\gamma \tag{15}$$

In the context of the SM, this allows us to extract the magnitude of the penguin, $|P|$ using the value of γ known independently from the global fit to the unitarity triangle. To the extent that the SM contribution to the exclusive final state can be calculated from QCD, admittedly a very challenging task, this can be compared to such a measured value of $|P|$.

In any case, a precise determination of the weak phase in some modes may also allow us to test the SM by feeding it into the isospin analysis discussed in Section V. In particular, the weak phase in the neutral D decay can allow us to determine the weak phase of the $\Delta I = \frac{3}{2}$ channel. Such a phase would be an indication of tree-like NP.

In this section we consider three methods to accomplish a weak phase measurement in D^0 decays by producing mixed initial states of D^0 and $\overline{D}^0$ mesons. First we consider the "conventional approach" of taking advantage of the natural mixing of the two flavor eigenstates, D^0 versus $\overline{D}^0$. Since the rate of this mixing is small compared to the D^0 lifetime, we consider using the much larger relative mixing in the B_d^0 and B_s^0 mesons if they subsequently decay to a D^0 meson. Finally we consider methods using entangled $D^0\overline{D}^0$ states produced at tau-charm factories where the meson pair arises form a ψ'' resonance and also using (super) B-factories where the pair arises in a two or three body

Decay	Suppressed Tree	Charged Final State	Favored	Total BR (10^{-3})
$D_s \to \pi^{(*)0} K^{(*)+}$	X	$[\rho^0 \to \pi^+\pi^-]K^+$	X	2.7 ± 0.05
		$[\rho^0 \to \pi^+\pi^-][K^{*+} \to \pi^+[K_s \to \pi^+\pi^-]]$	X	—
$D_s \to \phi^{(*)} K^{(*)+}$		$[\phi \to K^+K^-]K^+$		< 0.3
		$[\phi \to K^+K^-][K^{*+} \to \pi^+[K_s \to \pi^+\pi^-]]$		—
$D^+ \to \pi^{(*)+} \phi^{(*)}$	X	$\pi^+[\phi \to K^+K^-]$	X	2.65 ± 0.08
$D^+ \to K^{(*)+}\overline{K}^{(*)0}$		$K^+[K_s \to \pi^+\pi^-]$		1.98 ± 0.13
		$K^+[\overline{K}^{*0} \to K^+\pi^-]$		$2.45^{.09}_{.14}$
		$[K^{*+} \to \pi^+[K_s \to \pi^+\pi^-]][K_s \to \pi^+\pi^-]$		5.7 ± 2.3
		$[K^{*+} \to \pi^+[K_s \to \pi^+\pi^-]][\overline{K}^{*0} \to K^+\pi^-]$		—
$D^+ \to \pi^{(*)+}\pi^{(*)0}$		$\pi^+[\rho^0 \to \pi^+\pi^-]$		0.81 ± 0.15
$D^0 \to K^{(*)0}\overline{K}^{(*)0}$	XX	$[K_s \to \pi^+\pi^-][K_s \to \pi^+\pi^-]$	X	0.085 ± 0.014
		$[K^{*0} \to K^+\pi^-][K_s \to \pi^+\pi^-]$	X	< 0.2
		$[\overline{K}^{*0} \to K^-\pi^+][K_s \to \pi^+\pi^-]$	X	< 0.35
		$[K^{*0} \to K^+\pi^-][\overline{K}^{*0} \to \pi^+K^-]$	X	$.07 \pm 0.05$
$D^0 \to \pi^{(*)0}\pi^{(*)0}$	X	$[\rho^0 \to \pi^+\pi^-][\rho^0 \to \pi^+\pi^-]$	X	1.82 ± 0.10
$D^0 \to \pi^{(*)+}\pi^{(*)-}$		$\pi^+\pi^-$		$1.400 \pm .026$
$D^0 \to \phi^{(*)}\pi^{(*)0}$	X	$D^0 \to \phi\rho^0$	X	1.40 ± 0.12
$D^0 \to K^{(*)+}K^{(*)-}$		K^+K^-		$3.96 \pm .08$
		$[K^{*+} \to \pi^+[K_s \to \pi^+\pi^-]]K^-$		2.19 ± 0.1
		$K^+[K^{*-} \to \pi^-[K_s \to \pi^+\pi^-]]$		0.78 ± 0.06
		$[K^{*+} \to \pi^+[K_s \to \pi^+\pi^-]][K^{*-} \to \pi^-[K_s \to \pi^+\pi^-]]$		—

TABLE I: The singly Cabibbo suppressed decays of D mesons to two ground state are listed. Note that the notation $\pi^{(*)\pm}$ stands for π^+ or ρ^+; $\pi^{(*)0}$ stands for π^0, ρ^0 or ω^0; $\phi^{(*)}$ stands for ϕ or $\eta^{(\prime)}$ to the extent that $\eta^{(\prime)}$ is an $s\overline{s}$ state. For each group of decays, we have indicated whether the tree contribution is color suppressed with "X" and if it is both color and Zweig suppressed with "XX". The instances which lead to an all charged final state are listed. The favored column are decays where the tree is colored suppressed and the final state has an all charged final state indicated by "X". Where the branching ratios are known from [36] we have included it in the last column; this is the branching ratio including the subsequent decays to the final all charged state indicated.

decay of a B meson (e.g. $B \to D\overline{D}$ or $D\overline{D}K$). First, however, we quickly review the oscillation formalism for neutral mesons:

A. Oscillation Formalism

Let us consider a generic neutral flavored meson X (i.e. $X = K^0$, D^0, B_d or B_s). Defining the light eigenstate (X_L) with mass m_{XL} and width Γ_{XL} and heavy eigenstate (X_H) with mass m_{XH} and width Γ_{XH}, we have:

$$\begin{aligned} |X_L\rangle &= p^X |X\rangle + q^X |\overline{X}\rangle \\ |X_H\rangle &= p^X |X\rangle - q^X |\overline{X}\rangle \end{aligned} \tag{16}$$

Thus the flavor eigenstates evolve with time t_X according to

$$\begin{aligned} |X(t)\rangle_{phys} &= g_+^X |X\rangle - \frac{q^X}{p^X} g_-^X |\overline{X}\rangle \\ |\overline{X}(t)\rangle_{phys} &= g_+^X |\overline{X}\rangle - \frac{p^X}{q^X} g_-^X |X\rangle \end{aligned} \tag{17}$$

where the time dependent mixing coefficients $g_\pm^X$ are given by:

$$g_\pm^X = e^{-(im_{XH} + \frac{1}{2}\Gamma_{XH})t} \pm e^{-(im_{XL} + \frac{1}{2}\Gamma_{XL})t} \tag{18}$$

Let f be a final state which both X and $\overline{X}$ can decay to. If A_f^X is the amplitude for $X \to f$ and $\overline{A}_f^X$ is the amplitude for $\overline{X} \to f$ then the time dependent rates of X and $\overline{X}$ to f are:

$$\begin{aligned}
&\frac{d}{d\tau_X}\Gamma(X(t)\to f) = \\
&\frac{1}{2}e^{-\tau_X}\left[(C_y^X + C_x^X)|A_f^X|^2 + (C_y^X - C_x^X)\left|\frac{q^X}{p^X}\overline{A}_f^X\right|^2 + 2S_y^X Re\left(\frac{q^{X*}}{p^{X*}}A_f^X\overline{A}_f^{X*}\right) + 2S_x^X Im\left(\frac{q^{X*}}{p^{X*}}A_f^X\overline{A}_f^{X*}\right)\right] \\
&\frac{d}{d\tau_X}\Gamma(\overline{X}(t)\to f) = \\
&\frac{1}{2}e^{-\tau_X}\left[(C_y^X + C_x^X)|\overline{A}_f^X|^2 + (C_y^X - C_x^X)\left|\frac{p^X}{q^X}A_f^X\right|^2 + 2S_y^X Re\left(\frac{p^{X*}}{q^{X*}}A_f^{X*}\overline{A}_f^{X}\right) + 2S_x^X Im\left(\frac{p^{X*}}{q^{X*}}A_f^{X*}\overline{A}_f^{X}\right)\right]
\end{aligned} \quad (19)$$

Here $\Delta m_X = m_{XH} - m_{XL}$, $\Delta\Gamma_X = \Gamma_{XH} - \Gamma_{XL}$, $\tau_X = \Gamma_X t$, $x_X = \Delta m_X/\Gamma_X$, $y_X = \Delta\Gamma_X/(2\Gamma_X)$, $C_x^X = \cos(x_X\tau_X)$, $S_x^X = \sin(x_X\tau_X)$, $C_y^X = \cosh(y_X\tau_X)$, $S_y^X = \sinh(y_X\tau_X)$ and $z_X = x_X - iy_X$.

In the case of D mesons, both x_D and $y_D \le O(10^{-2})$, so we will expand observables to first order in x_D and y_D.

In this limit, the above time dependent rate becomes:

$$\begin{aligned}
\frac{d}{d\tau_X}\Gamma(X(t)\to f) &= e^{-\tau_X}\left[|A_f^X|^2 + \tau Re\left[-iz^* A_f^X\overline{A}_f^{X*}\frac{q^*}{p^*}\right]\right] + O(x^2,y^2) \\
\frac{d}{d\tau_X}\Gamma(\overline{X}(t)\to f) &= e^{-\tau_X}\left[|\overline{A}_f^X|^2 + \tau Re\left[-iz^* A_f^{X*}\overline{A}_f^{X}\frac{p^*}{q^*}\right]\right] + O(x^2,y^2)
\end{aligned} \quad (20)$$

In some of the examples below we will consider the time integrated effect of oscillation. To first order in x_X, y_X this can be accomplished by replacing the decay amplitudes with "effective" decay amplitudes:

$$\begin{aligned}
B_f^X &= A_f^X + \frac{i}{2}\overline{A}_f^X\left(\frac{q^X}{p^X}\right)z \\
\overline{B}_f^X &= \overline{A}_f^X + \frac{i}{2}A_f^X\left(\frac{q^X}{p^X}\right)z
\end{aligned} \quad (21)$$

The time integrated rate for a D^0 meson to decay to f is given, up to first order in x_X and y_X, by using this effective amplitude "without oscillation".

Thus, for instance,

$$\begin{aligned}
\int_0^\infty d\Gamma(X\to f)\, d\tau &= |B_f^X|^2 + O(x^2,y^2) \\
\int_0^\infty d\Gamma(\overline{X}\to f)\, d\tau &= |\overline{B}_f^X|^2 + O(x^2,y^2)
\end{aligned} \quad (22)$$

B. Weak Phases from $D^0/\overline{D}^0$ Oscillation

As discussed in [41, 42] in D^0 decay to a given final state one must consider both direct CP violation and indirect CP violation due to D^0 oscillation. Conversely, assuming that the oscillation parameters are known from separate studies, we can use oscillation extract the phase between A_f^X and $\overline{A}_f^X$. To do this, it is necessary to observe the time dependence of the decays.

From eqn. 20 if we know x_D, y_D, p^D and q^D we see that the constant term gives the magnitudes of the amplitudes $|A_f^D|$ and $|\overline{A}_f^D|$. The slope of the decay rate gives the phase between these two amplitudes. If f is self conjugate like $\pi^+\pi^-$ then such a phase difference will be CP-odd. If f is not self conjugate, such as $\rho^+\pi^-$ then the phase will be a combination of CP-odd and CP-even phase differences. If both x_D and y_D are non-zero, then the phase can be determined in this way without ambiguity. If one of these is zero, then there is a two-fold ambiguity in the phase determination.

C. Weak Phases from $B_q^0/\overline{B}_q^0$ Oscillation

Another way to accomplish the measurement of the relative phase is to look at a two body decay of a neutral B-meson, B_q for $q = d, s$, to a neutral D-meson where the D-meson subsequently decays to the final state f. If we observe this overall reaction $B \to M^0[D^0 \to f]$ (where M^0 is a self conjugate neutral meson and B is either B_d or B_s) as a function of the time of the B_q decay then the D state involved in the second decay will generally be a mixture of the flavor eigenstates.

Of course, once the D-meson is spawned, it will oscillate as described above. In the following we will assume that only the B-meson decay time is observed and therefore the D-meson decay time is integrated over.

Let us denote by T_{DB} the amplitude for $B \to M^0 D$, $T_{\overline{D}B}$ the amplitude for $B \to M^0\overline{D}$, $T_{D\overline{B}}$ the amplitude for $\overline{B} \to M^0 D$ and $T_{\overline{DB}}$ the amplitude for $\overline{B} \to M^0\overline{D}$ and thus we can define the effective amplitudes for B and $\overline{B}$ cascading down to the final state f Using the formalism in eqn. 21 we can define the effective amplitudes:

$$\begin{aligned} D_f^B &= B_f^D T_{DB} + \overline{B}_f^D T_{\overline{D}B} \\ \overline{D}_f^B &= B_f^D T_{D\overline{B}} + \overline{B}_f^D T_{\overline{DB}} \end{aligned} \tag{23}$$

Thus the time dependent decay rate integrated over the D-meson decay time as a function of the B-meson decay time is given by eqn. 19:

$$\begin{aligned} &\frac{d}{d\tau_B}\Gamma(B(t_B) \to M[D \to f]) = \\ &\frac{1}{2}e^{-\tau_B}\left[(C_y^B + C_x^B)|D_f^B|^2 + (C_y^B - C_x^B)\left|\frac{q^B}{p^B}\overline{D}_f^B\right|^2 + 2S_y^B Re\left(\frac{q^{B*}}{p^{B*}}D_f^B\overline{D}_f^{B*}\right) + 2S_x^B Im\left(\frac{q^{B*}}{p^{B*}}D_f^B\overline{D}_f^{B*}\right)\right] \\ &\frac{d}{d\tau_B}\Gamma(\overline{B}(t_B) \to M[D \to f]) = \\ &\frac{1}{2}e^{-\tau_B}\left[(C_y^B + C_x^B)|\overline{D}_f^B|^2 + (C_y^B - C_x^B)\left|\frac{p^B}{q^B}D_f^B\right|^2 + 2S_y^B Re\left(\frac{p^{B*}}{q^{B*}}D_f^{B*}\overline{D}_f^B\right) + 2S_x^B Im\left(\frac{p^{B*}}{q^{B*}}D_f^{B*}\overline{D}_f^B\right)\right] \end{aligned} \tag{24}$$

From the above equation, assuming that x_B, y_B and p^B/q^B are known then the magnitudes and relative phase of D_f^B and $\overline{D}_f^B$ can be determined. Assuming that T_{ij} is also known, then by inverting Eqn. 23 we determine B_f^D and $\overline{B}_f^D$. As in the last section, we can then invert the relations contained in Eqn. 21 to determine the magnitudes and relative phases of A_f^D and $\overline{A}_f^D$. In most cases the amplitudes $T_{\overline{D}B} \approx \pm T_{D\overline{B}}$ will dominate over T_{DB} and $T_{\overline{DB}}$ and since $\{B_f^D, \overline{B}_f^D\}$, differs from $\{A_f^D, \overline{A}_f^D\}$, by $O(10^{-2})$ the phase between D_f^B and $\overline{D}_f^B$ will, to a good approximation, be the negative of the phase between A_f^D and $\overline{A}_f^D$.

Let us consider some particular cases of the parent $B \to MD$ decay. In the case of B_d some candidates are $B_d \to \pi^0\overline{D}^0$ (Br=$2.61 \pm 0.24 \times 10^{-4}$) and $B_d \to \rho^0\overline{D}^0$ (Br=$3.2 \pm 0.5 \times 10^{-4}$). The latter is probably easier to observe since ρ^0 decays to $\pi^+\pi^-$. Indeed if the final state of the D^0 decay is either $\pi^+\pi^-$ or $\rho^0\rho^0$ then the entire event has an all charged final state. Other decays of this type are $B_d \to \eta\overline{D}^0$ (Br=$2.02 \pm 0.35 \times 10^{-4}$), $B_d \to \eta'\overline{D}^0$ (Br=$1.25 \pm 0.23 \times 10^{-4}$) and $B_d \to \omega\overline{D}^0$ (Br=$2.59 \pm 0.3 \times 10^{-4}$). In principle the results from these modes can be combined (taking into account the CP of the final state). In this case we could have an aggregate branching ratio of $\sim 10^{-3}$. We can also consider $\overline{D}^{0*}$ instead of $\overline{D}^0$ and that can augment the effective branching ratio.

Another choice is to consider decays such as $B_d \to K_s\overline{D}$ (Br=5.2×10^{-5}) and related modes but these are an order of magnitude smaller in branching ratio due to Cabibbo suppression.

It is also possible to start with a B_s state. The analogous decays are $B_s \to K_s D$ and related processes which likely have roughly the same branching ratios. These would include $B_s \to K_s\overline{D}^0$ and $B_s \to K^{*0}\overline{D}^0$ with branching ratios at the $10^{-4} - 10^{-3}$ level (note the K^* would need to decay to $K_s\pi^0$ to collaps the B_s flavor wave function) and decays such as $B_s \to \phi\overline{D}^0$ at the $10^{-5} - 10^{-4}$ level.

D. Correlations at Charm and B Factories

Let us now consider the case of a $D^0\overline{D}^0$ pair which is initially in a single, correlated, quantum state. Let us arbitrarily label the mesons D_1 and D_2 and consider reactions where $D_1 \to f$; $D_2 \to g$ where f is the state of interest and g is an "index" decay (which needs to be a decay state of both D^0 and $\overline{D}^0$), for instance $f = \pi^+\pi^-$ and $g = K_s\pi^0$ where the weak phase of $\pi^+\pi^-$ is to be measured and it is assumed that the weak phase in $K_s\pi^0$ is small and known (i.e. just from K_s).

In such a scenario, the initial wave function together with the observation of $D_2 \to g$ determines the wave function of the D_1 state as a mixture of the flavor eigenstates. In this way we are able to observe the interference of D^0 and $\overline{D}^0$ decay amplitudes to f [43, 44].

Starting with the wave function of the meson pair:

$$\Psi = a|D_1> |\overline{D}_2> + \overline{a}|\overline{D}_1> |D_2> \tag{25}$$

where $|a|^2 + |\overline{a}|^2 = 1$, the amplitude for the combined decay $(D_1 \to f)(D_1 \to g)$ is therefore

$$\begin{aligned} A_{fg} &= aA_f\overline{A}_g + \overline{a}\overline{A}_f A_g \\ |A_{fg}|^2 &= \frac{1}{2}(|A_f|^2|\overline{A}_g|^2 + |\overline{A}_f|^2|A_g|^2) + \frac{1}{2}(|a|^2 - |\overline{a}|^2)(|A_f|^2|\overline{A}_g|^2 - |\overline{A}_f|^2|A_g|^2) + 2Re(a\overline{a}^* A_f\overline{A}_f^* A_g^*\overline{A}_g). \end{aligned} \tag{26}$$

Thus if $|A_f|$, $|\overline{A}_f|$, A_g, $\overline{A}_g$, θ and δ are known, then the phase between A_f and $\overline{A}_f$ can be determined.

This equation takes into account only the entanglement of the initial state, we can also take into account the time integrated neutral D oscillations by integrating $|A_{fg}|^2$ to first order in x, y. As above, this is equivalent to replacing $A_{f,g}$ in Eqn. 26 with the effective amplitudes $B_{f,g}$ given by Eqn. 21.

The conceptually simplest example is applying this at a tau-charm factory using the method of [43, 44]. In this case, the D^0 pair arises from the decay of the $\psi(3770)$ and so in the initial state,

$$a = +\frac{1}{\sqrt{2}} \qquad \overline{a} = -\frac{1}{\sqrt{2}}.$$

An evenly mixed D_1 state will thus arise when $|A_g| \approx |\overline{A}_g|$. As an example, if we take $g = K_s\pi^0$ then $A_g = -\overline{A}_g$ if, as the SM predicts, there is no CP violation in this pure tree decay mode except for the well understood $O(10^{-3})$ CP violation in the mixing of the K_s. (CP violation in D decay to states which contain K_s has been observed in the related decay $D^+ \to K_s\pi^+$ by BELLE [45] and has been shown to be consistent with CP violation only due to the well understood mixing in the neutral kaon.)

This method may be generalized somewhat to the case where g is a three body decay such as $K_s\pi^+\pi^-$. Here, the decay amplitude is a function of the kinematic variables. In this case we can specify the kinematics by the variables $E_\pm$ being the energies of the $\pi^\pm$ in the rest frame of the D^0. The amplitudes A_g and $\overline{A}_g$ are functions of these variables: $A_g(E^+, E^-)$ and $\overline{A}_g(E^+, E^-)$. If the decay to g is CP-invariant then the relation $\overline{A}_g(E^+, E^-) = A_g(E^-, E^+)$. If we assume that $A_g(E^+, E^-)$ and $\overline{A}_g(E^+, E^-)$ are known from other studies then eqn. 26 can be use to find the phase between A_f and $\overline{A}_f$.

Another potential way to generate correlated neutral D-meson pairs is at a B factory through decays such as $B^+ \to D^0\overline{D}^0K^+$ (Br=$2.10 \pm 0.26 \times 10^{-3}$). More generally, it should be possible to adapt this analysis to the decays $B^+ \to D^{*0}\overline{D}^0K^+$ ($Br = 4.7 \pm 1.0 \times 10^{-3}$), $B^+ \to D^{*0}\overline{D}^{*0}K^+$ ($Br = 5.3 \pm 1.6 \times 10^{-3}$) and $B^+ \to D^0\overline{D}^{*0}K^+$ (BR not yet known) which may increase the statistics by a factor of ~ 5.

Observing the Dalitz plot of $B^+ \to D^0\overline{D}^0K^+$ decay and fitting it to a resonance+background will give a model for the phase of the decay amplitude as a function of the Dalitz plot variables.

Let us take the Dalitz plot variables to be E_D and $\overline{E}_D$, the energies of the D^0 and $\overline{D}^0$ in the B^+ frame respectively so that the decay amplitude will have the dependency $A(E_D, \overline{E}_D)$. Let E_f be the energy in the B^+ frame of state f and E_g be the energy of state g. The wave function of the D-meson pair is therefore given in terms of E_f and E_g by (note $f \leftrightarrow g$ between the two equations):

$$\begin{aligned} a(E_f, E_g) &= A(E_f, E_g)/\sqrt{|A(E_f, E_g)|^2 + |A(E_g, E_f)|^2} \\ \overline{a}(E_f, E_g) &= A(E_g, E_f)/\sqrt{|A(E_f, E_g)|^2 + |A(E_g, E_f)|^2} \end{aligned} \tag{27}$$

This is because there is interference between the case where $D^0 \to f$ with $\overline{D}^0 \to g$ and $\overline{D}^0 \to f$ with $D^0 \to g$. Using Dalitz plot phases in this way is similar to a method used by BaBar to find the phase γ in the B-meson decay to $D^0 K$ where the D meson subsequently decays to 3π [46].

V. ISOSPIN DECOMPOSITIONS

Since isospin is a very good symmetry of strong interactions, conclusions reached based on isospin alone should hold quite accurately in spite of some theoretical uncertainties due to hadronic interactions. In a recent application of isospin to D decays [15] it is argued that although generally isospin breaking is $O(1\%)$ the isospin breaking contribution to CP violation should be second order in the isospin breaking parameter.

The SM predicts that there is no CP violation in the $\Delta I = \frac{3}{2}$ channel because the contribution to this channel is only through the tree graph $c \to d\bar{d}u$ while the QCD penguin which has the CP violating phase is pure $\Delta I = \frac{1}{2}$. In principle the Electroweak penguin could introduce CP violation into the $\Delta I = \frac{3}{2}$ channel but, as discussed below, this amplitude is negligibly small in D decays.

In principle, at higher order in the SM, the electro-weak penguin (EWP) graphs could also contribute to CP violation in the $\Delta I = \frac{3}{2}$ channel. We can see, however, such contributions will be very small as follows: First, one expects that these will be suppressed compared to the QCD penguin by a factor of $\alpha_W/\alpha_s \sim O(1\%)$. In the analogous case of B physics, the EW penguins are thought to be large in part due to enhancement $\propto m_t$ which is not the case in charm decays. For example, in the decay $B \to K^+\pi^-$, $|A_{CP}| = 9\%$ so if we assume that all of this CP violation is due to EWP interfering with the tree graph, then we can crudely estimate the corresponding EWP contribution to the asymmetry in D decay as follows:

$$\begin{aligned} A_{CP}^{EWP}(D \to \pi^+\pi^-) &\approx \frac{(EWP(D))\,(Penguin(B))}{(Tree(D))\,(EWP(B))} A_{CP}(B \to \pi^+ K^-) \\ &\approx \frac{Penguin(B)}{Tree(B)} \frac{(|V_{cb}||V_{ub}|m_b)(|V_{ub}||V_{us}|)}{(|V_{ud}||V_{cd}|)(|V_{tb}||V_{ts}|m_t)} A_{CP}(B \to \pi^+ K^-) \\ &\approx \frac{Penguin(B)}{Tree(B)} |V_{ub}|^2 \frac{m_b}{m_t} \sim O(10^{-5}) \end{aligned} \tag{28}$$

which suggests even a smaller contribution. There are a number of channels where we can directly test the premise that isospin is a good symmetry in D-meson decays to two body final states. As we discuss below, the relative phases in decays to $\rho\rho$ provide a test of isospin conservation.

Turning now to the isospin decomposition of SCS D-meson decays, we proceed in analogy to previous work in the case of $B \to \pi\pi$ and related processes [47][48]. In our expansion we will adopt a notation similar to [48]. For each particular final state we will denote the isospin amplitude by $A^f_{\Delta I\ I} = A^f_{T:\Delta I\ I} + A^f_{P:\Delta I\ I}$ which indicates the amplitude for a transition through an effective Hamiltonian with isospin change ΔI leading to a final state of type f with total isospin I. The right hand side indicates the further decomposition of the given amplitude into tree and penguin contributions respectively. Likewise the notation $A^f_{ij} = A^f_{T:ij} + A^f_{P:ij}$ where $i, j \in \{+-0\}$ indicates the amplitude for a decay with the indicated charge distribution. The corresponding amplitudes for $\overline{D}$ decay are indicated by $\overline{A}$.

Using this notation, we find for the $\pi\pi$ final state:

$$\begin{aligned} A^{\pi\pi}_{+0} &= \frac{\sqrt{3}}{2} A^{\pi\pi}_{\frac{3}{2},2} \\ A^{\pi\pi}_{+-} &= \frac{1}{\sqrt{6}} A^{\pi\pi}_{\frac{3}{2},2} + \frac{1}{\sqrt{3}} A^{\pi\pi}_{\frac{1}{2},0} \\ A^{\pi\pi}_{00} &= \frac{1}{\sqrt{3}} A^{\pi\pi}_{\frac{3}{2},2} - \frac{1}{\sqrt{6}} A^{\pi\pi}_{\frac{1}{2},0} \end{aligned} \tag{29}$$

with the analogous relations also apply for the charge conjugate amplitudes.

This leads to the following "isospin triangle" relationships:

$$\frac{1}{\sqrt{2}} A^{\pi\pi}_{+-} + A^{\pi\pi}_{00} - A^{\pi\pi}_{+0} = 0 = \frac{1}{\sqrt{2}} \overline{A}^{\pi\pi}_{+-} + \overline{A}^{\pi\pi}_{00} - \overline{A}^{\pi\pi}_{-0} \tag{30}$$

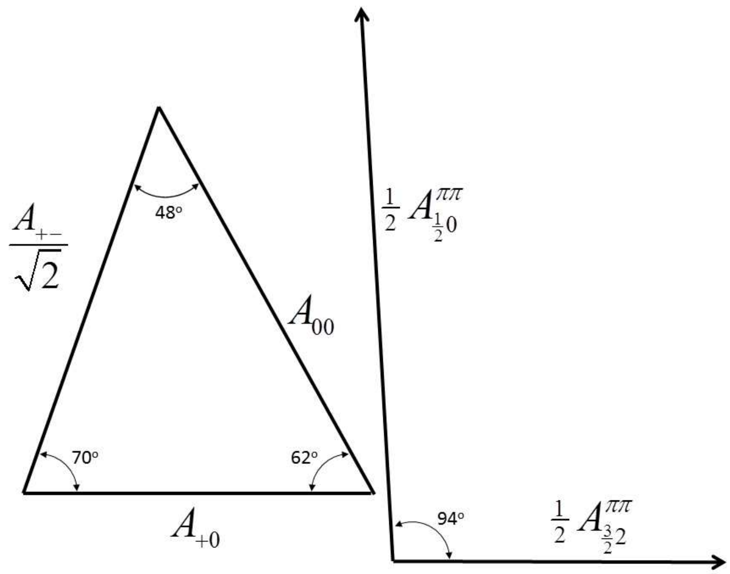

FIG. 4: A sketch of the isospin triangle given in eqn. 30 using the central value for the branching ratios of the three modes. The two vectors in the sketch are proportional to the isospin 2 and isospin 0 amplitudes as shown where the phase of the isospin 2 amplitude is arbitrarily taken to be real.

Figure 4 shows a sketch of such a triangle where we use the central values for the branching ratios involved. In the sketch we also show $A^{\pi\pi}_{\frac{1}{2},0}$ and $A^{\pi\pi}_{\frac{3}{2},0}$.

The case of the KK final state can also be expanded in a similar way. The resulting relations are:

$$\begin{aligned}
A^{KK}_{+0} &= -\frac{1}{2}A^{KK}_{\frac{3}{2},1} + A^{KK}_{\frac{1}{2},1} \\
A^{KK}_{+-} &= \frac{1}{2}A^{KK}_{\frac{3}{2},1} + \frac{1}{2}A^{KK}_{\frac{1}{2},1} + \frac{1}{2}A^{KK}_{\frac{1}{2},0} \\
A^{KK}_{00} &= \frac{1}{2}A^{KK}_{\frac{3}{2},1} + \frac{1}{2}A^{KK}_{\frac{1}{2},1} - \frac{1}{2}A^{KK}_{\frac{1}{2},0}
\end{aligned} \tag{31}$$

In this case there are three isospin amplitudes determining three decay amplitudes so we cannot construct a triangle relation such as Eqn. 30.

In the case of the $\pi\pi$ final state, we can see from Figure 4 that the two isospin amplitudes have a strong phase between them due to rescattering. In particular this illustrates the failure of color suppression in this system; for color suppression to be realized, the two amplitudes must cancel, thus be colinear in the complex plane and have magnitudes related by $\sqrt{2}A^{\pi\pi}_{\frac{3}{2},2} = A^{\pi\pi}_{\frac{1}{2},0}$.

In contrast for the K_sK_s case, evidentally there is considerable suppression of the branching ratio. This makes sense on the quark level since not only is the decay color suppressed but it is also Zweig suppressed. For this to happen, the isospin 1 tree amplitude and the isospin 0 tree amplitude must cancel fairly well: $\frac{1}{2}A^{KK}_{T:\frac{3}{2},1} + \frac{1}{2}A^{KK}_{T:\frac{1}{2},1} \approx \frac{1}{2}A^{KK}_{T:\frac{1}{2},0}$. Since the dominant tree amplitude is suppressed, this suggests the possibility that A_{CP} in $D^0 \to K_sK_s$ could be enhanced compared to the K^+K^- case at the expense of the total rate.

The $\rho\rho$ final states have the same form as $\pi\pi$ except that there are three polarization states which can arise from the decay of a scalar, $A_{\|}$, $A_{\perp}$ and A_{ℓ}. There will thus be a separate set of isospin amplitudes so the analog to the above decomposition is:

$$
\begin{aligned}
A_{+0}^{\rho\rho(i)} &= \frac{\sqrt{3}}{2} A_{\frac{3}{2},2}^{\rho\rho(i)} \\
A_{+-}^{\rho\rho(i)} &= \frac{1}{\sqrt{6}} A_{\frac{3}{2},2}^{\rho\rho(i)} + \frac{1}{\sqrt{3}} A_{\frac{1}{2},0}^{\rho\rho(i)} \\
A_{00}^{\rho\rho(i)} &= \frac{1}{\sqrt{3}} A_{\frac{3}{2},2}^{\rho\rho(i)} - \frac{1}{\sqrt{6}} A_{\frac{1}{2},0}^{\rho\rho(i)}
\end{aligned} \tag{32}
$$

where $i \in \{\perp, \|, \ell\}$ indexes the polarization state.

For each polarization state we therefore also have an isospin triangle relation:

$$
\frac{1}{\sqrt{2}} A_{+-}^{\rho\rho(i)} + A_{00}^{\rho\rho(i)} - A_{+0}^{\rho\rho(i)} = 0 = \frac{1}{\sqrt{2}} \overline{A}_{+-}^{\rho\rho(i)} + \overline{A}_{00}^{\rho\rho(i)} - \overline{A}_{-0}^{\rho\rho(i)} \tag{33}
$$

For each $\rho\rho$ final state, an angular analysis [37] provides the magnitude of the three polarization amplitudes and the cosine of the phase angle between them. Thus (up to a 2 fold ambiguity) the relative phases between these three amplitudes can be determined. The relation (Eqn. 33) gives the phase between the three amplitudes with the same charge distribution in terms of their magnitudes (again up to a 2 fold ambiguity). Combining the two kinds of information, we have 18 phase differences for 9 amplitudes in D-decays (Taking into account an overall phase, the system is overdetermined by 8 degrees of freedom) which checks the validity of Eqn. 33. Since this relation was derived using isospin conservation, the validity of this symmetry is thus quantified.

For the $\rho\pi$ final state the decomposition is:

$$
\begin{aligned}
A_{+0}^{\rho\pi} &= \frac{\sqrt{3}}{\sqrt{8}} A_{\frac{3}{2},2}^{\rho\pi} - \frac{1}{\sqrt{8}} A_{\frac{3}{2},1}^{\rho\pi} + \frac{1}{\sqrt{2}} A_{\frac{1}{2},1}^{\rho\pi} \\
A_{0+}^{\rho\pi} &= \frac{\sqrt{3}}{\sqrt{8}} A_{\frac{3}{2},2}^{\rho\pi} + \frac{1}{\sqrt{8}} A_{\frac{3}{2},1}^{\rho\pi} - \frac{1}{\sqrt{2}} A_{\frac{1}{2},1}^{\rho\pi} \\
A_{+-}^{\rho\pi} &= \frac{1}{\sqrt{12}} A_{\frac{3}{2},2}^{\rho\pi} + \frac{1}{2} A_{\frac{3}{2},1}^{\rho\pi} + \frac{1}{2} A_{\frac{1}{2},1}^{\rho\pi} + \frac{1}{\sqrt{6}} A_{\frac{1}{2},0}^{\rho\pi} \\
A_{-+}^{\rho\pi} &= \frac{1}{\sqrt{12}} A_{\frac{3}{2},2}^{\rho\pi} - \frac{1}{2} A_{\frac{3}{2},1}^{\rho\pi} - \frac{1}{2} A_{\frac{1}{2},1}^{\rho\pi} + \frac{1}{\sqrt{6}} A_{\frac{1}{2},0}^{\rho\pi} \\
A_{00}^{\rho\pi} &= \frac{1}{\sqrt{3}} A_{\frac{3}{2},2}^{\rho\pi} - \frac{1}{\sqrt{6}} A_{\frac{1}{2},0}^{\rho\pi}
\end{aligned} \tag{34}
$$

which in turn leads to the following pentagonal isospin relationships:

$$
\begin{aligned}
\sqrt{3}\ A_{\frac{3}{2},2}^{\rho\pi} &= \sqrt{2}(A_{+0}^{\rho\pi} + A_{0+}^{\rho\pi}) = A_{+-}^{\rho\pi} + A_{-+}^{\rho\pi} + 2A_{00}^{\rho\pi} \\
\sqrt{3}\ \overline{A}_{\frac{3}{2},2}^{\rho\pi} &= \sqrt{2}(\overline{A}_{-0}^{\rho\pi} + \overline{A}_{0-}^{\rho\pi}) = \overline{A}_{-+}^{\rho\pi} + \overline{A}_{+-}^{\rho\pi} + 2\overline{A}_{00}^{\rho\pi}
\end{aligned} \tag{35}
$$

also the $\Delta I = 3/2$ contribution to the $I = 1$ final state follows from the relation:

$$
\begin{aligned}
3\ A_{\frac{3}{2},1}^{\rho\pi} &= \sqrt{2}(A_{0+}^{\rho\pi} - A_{+0}^{\rho\pi}) + 2(A_{+-}^{\rho\pi} - A_{-+}^{\rho\pi}) \\
3\ \overline{A}_{\frac{3}{2},1}^{\rho\pi} &= \sqrt{2}(\overline{A}_{0+}^{\rho\pi} - \overline{A}_{+0}^{\rho\pi}) + 2(\overline{A}_{+-}^{\rho\pi} - \overline{A}_{-+}^{\rho\pi})
\end{aligned} \tag{36}
$$

In the case of the decay $D_s \to \pi K^*$ the isospin decomposition of the amplitudes is:

$$
\begin{aligned}
A_{+0}^{\pi K^*} &= \frac{1}{\sqrt{3}} A_{\frac{3}{2}}^{\pi K^*} + \frac{\sqrt{2}}{\sqrt{3}} A_{\frac{1}{2}}^{\pi K^*} \\
A_{0+}^{\pi K^*} &= \frac{\sqrt{2}}{\sqrt{3}} A_{\frac{3}{2}}^{\pi K^*} - \frac{1}{\sqrt{3}} A_{\frac{1}{2}}^{\pi K^*} \\
\text{Thus,}\quad \sqrt{3} A_{\frac{3}{2}}^{\pi K^*} &= A_{+0}^{\pi K^*} + \sqrt{2} A_{0+}^{\pi K^*}
\end{aligned} \tag{37}
$$

In this case, the two decay amplitudes depend on two isospin amplitudes so there is no isospin triangle relation as in the case of $\pi\pi$ and $\rho\pi$.

A. Phases in Dalitz Plots

In the two body decays $D \to \rho\pi$ and $D_s \to K^*\pi$ the vectors decay in turn to two pseudoscalars, $\rho \to \pi\pi$ and $K^* \to K\pi$. The final states are therefore three body Dalitz decays[49, 50]. The same three scalar final state will, in general, receive contributions from a number of different pseudo two body channels. For example in the case of $D_s \to K^*\pi$, the two charge distributions will contribute to the same three body final state, in particular $D_s \to K^{*+}\pi^0 \to K^0\pi^+\pi^0$ and $D_s \to K^{*0}\pi^+ \to K^0\pi^0\pi^+$. Thus the $K^0\pi^0\pi^+$ final state receives contributions from both the $K^{*0}\pi^+$ and $K^{*+}\pi^0$ channels. A fit to the the distribution in the Dalitz plot variables will therefore determine both the magnitudes of the two body amplitudes and also the relative phase between them as well as other channels which contribute to this final state such as $K^0\rho^+$. Note that the other decay of the K^* in the above will not involve interference between these two channels, in particular $D_s \to K^{*+}\pi^0 \to K^+\pi^0\pi^0$ and $D_s \to K^{*0}\pi^+ \to K^+\pi^-\pi^+$.

The same situation also applies to $D^0 \to \rho\pi$ which leads to the final state $\pi^+\pi^-\pi^0$. In this case the pseudo two body channels $\rho^0\pi^0$, $\rho^+\pi^-$ and $\rho^-\pi^+$ all contribute so in fitting the Dalitz plot one obtains the magnitude and relative phases of each of these channels.

B. Standard Model Tests using Isospin

The main test of the SM origin for CP violation in hadronic D decays which can be accomplished using isospin analysis is to test the SM prediction that the tree graph which is the only contribution to the $\Delta I = 3/2$ Hamiltonian, has no phase in the Wolfenstein phase convention. Thus, assuming EWP are negligible, any CP violation in phase or magnitude is contained in the $\Delta I = 1/2$ component which receives contributions both from the tree and the penguin.

In each system of decays there are therefore two kinds of tests which, in principle, can be performed.

1. The magnitude of the $\Delta I = 3/2$ transition amplitude is the same for the decay and its charge conjugate.

2. The phase of the $\Delta I = 3/2$ transition amplitude is the same for the decay and its charge conjugate.

In the $\pi\pi$ final state, the system is sufficiently simple to allow us to cleanly extract three isospin related CP asymmetries. To fully characterize CP violation in this system, a fourth quantity must be determined by a phase measurement of the type described in Section IV.

Let us denote by δ^f_{ij} the partial rate difference for final state f with charge distribution ij, i.e. $\delta^f_{ij} = |A^f_{ij}|^2 - |\overline{A}^f_{ij}|^2$. Likewise for the isospin amplitudes denote:

$$\begin{aligned}
\delta^f_{\Delta I\ I} &= |A^f_{\Delta I\ I}|^2 - |\overline{A}^f_{\Delta I\ I}|^2 \\
\delta^f_{[\Delta I\ I;\Delta J\ J]} &= Re\left(A^f_{\Delta I\ I}A^{f*}_{\Delta J\ J} - \overline{A}^f_{\Delta I\ I}\overline{A}^{f*}_{\Delta J\ J}\right)
\end{aligned} \tag{38}$$

Using this notation, Eqn. 29 implies that the CP violation in the $D \to \pi\pi$ decays can be rewritten as:

$$\begin{aligned}
\delta^{\pi\pi}_{+0} &= \frac{3}{4}\delta^{\pi\pi}_{\frac{3}{2}2} \\
\delta^{\pi\pi}_{+-} &= \frac{1}{6}\delta^{\pi\pi}_{\frac{3}{2}2} + \frac{1}{3}\delta^{\pi\pi}_{\frac{1}{2}0} + \frac{1}{3}\sqrt{2}\delta^{\pi\pi}_{\frac{3}{2}2;\frac{1}{2}0} \\
\delta^{\pi\pi}_{00} &= \frac{1}{3}\delta^{\pi\pi}_{\frac{3}{2}2} + \frac{1}{6}\delta^{\pi\pi}_{\frac{1}{2}0} - \frac{1}{3}\sqrt{2}\delta^{\pi\pi}_{\frac{3}{2}2;\frac{1}{2}0}
\end{aligned} \tag{39}$$

Since this gives each of the observed partial rate differences in terms of three different CP violating underlying isospin quantities, we can invert these and obtain:

$$\begin{aligned}
\delta^{\pi\pi}_{\frac{3}{2}2} &= \frac{4}{3}\delta^{\pi\pi}_{+0} \\
\delta^{\pi\pi}_{\frac{1}{2}0} &= 2\delta^{\pi\pi}_{+-} + 2\delta^{\pi\pi}_{00} - \frac{4}{3}\delta^{\pi\pi}_{+0} \\
\frac{1}{\sqrt{2}}\delta^{\pi\pi}_{\frac{3}{2}2;\frac{1}{2}0} &= \frac{1}{3}\delta^{\pi\pi}_{+0} + \frac{1}{2}\delta^{\pi\pi}_{+-} - \frac{1}{2}\delta^{\pi\pi}_{00}
\end{aligned} \tag{40}$$

As discussed in [15] the first expression for $\delta^{\pi\pi}_{\frac{3}{2}2}$ implies, $\delta^{\pi\pi}_{\frac{3}{2}2}=0$ is a test of type (1) due to the evident fact that the decay to $\pi^+\pi^0$ is governed only by the $\Delta I=\frac{3}{2}$ Hamiltonian.

The other two combinations indicate different features of CP violation in the $\Delta I=\frac{1}{2}$ channel which could be entirely due to SM physics. As discussed in [15], for $\delta^{\pi\pi}_{\frac{1}{2}0}$ to be non-zero requires that there are two contributions to this isospin channel which have different strong phases and also different weak phases. This would generally be expected to be the case in the SM since both tree and penguin contribute to $\Delta I=\frac{1}{2}$. It could, however, happen that $\delta^{\pi\pi}_{\frac{1}{2}0}$ is small due to the strong phase difference between the two contributions to $\Delta I=\frac{1}{2}$ being small but in this case the quantity $\delta^{\pi\pi}_{\frac{3}{2}2;\frac{1}{2}0}$ could be non-zero due to strong and weak phase difference between the two different isospin channels.

To make this clear, consider, for example, what happens if $\delta^{\pi\pi}_{\frac{3}{2}2}=\delta^{\pi\pi}_{\frac{1}{2}0}=0$ but $\delta^{\pi\pi}_{\frac{3}{2}2;\frac{1}{2}0}\neq 0$. This would imply first that $|A^{\pi\pi}_{\frac{3}{2}2}|=|\overline{A}^{\pi\pi}_{\frac{3}{2}2}|$ and $|A^{\pi\pi}_{\frac{1}{2}2}|=|\overline{A}^{\pi\pi}_{\frac{1}{2}2}|$ but that the phase between $A^{\pi\pi}_{\frac{3}{2}2}$ and $A^{\pi\pi}_{\frac{1}{2}0}$ is different than the phase between $\overline{A}^{\pi\pi}_{\frac{3}{2}2}$ and $\overline{A}^{\pi\pi}_{\frac{1}{2}0}$ resulting from a different weak phase between the two isospin channels.

The measurement of the phase difference between either of the neutral amplitudes and their charge conjugates, i.e. $A^{\pi\pi}_{+-}$ versus $\overline{A}^{\pi\pi}_{+-}$ or $A^{\pi\pi}_{00}$ versus $\overline{A}^{\pi\pi}_{00}$ using the methods in Section IV allows the complete determination of all the amplitudes and therefore all the CP violation in this system. This follows from relation eqn. 30 which implies that the three amplitudes form a triangle in the complex plane. The phase between $A^{\pi\pi}_{+0}$ and either of the neutral modes is therefore determined (up to a 2 fold ambiguity) and so the phase of $A^{\pi\pi}_{+0}$ is known. The same is also true for the charge conjugate amplitudes so ultimately the weak phase between $A^{\pi\pi}_{+0}$ and $\overline{A}^{\pi\pi}_{-0}$ is determined (up to a four fold ambiguity). This then is a test of the SM of type (2).

Note that if the phase difference with the conjugates is measured for both $\pi^+\pi^-$ and $\pi^0\pi^0$ final states, then there is a consistency check for the isospin relations because the isospin triangle for D^0 decay fixes the phase between $A^{\pi\pi}_{+-}$ and $A^{\pi\pi}_{00}$ while the the isospin triangle for $\bar{D}^0$ decay fixes the phase between $\overline{A}^{\pi\pi}_{+-}$ and $\overline{A}^{\pi\pi}_{00}$. In addition, having both phase measurements will resolve the four fold ambiguity with respect to the orientation of the isospin triangles. Of course measuring the weak phase directly with the $\pi^0\pi^0$ final state using the methods described above will likely be experimentally difficult.

The same discussion also applies to each polarization of the final state. the $\rho\rho$ final state. Because all of the relative phases of the 9 $D\to\rho\rho$ amplitudes can be measured as discussed above (and likewise for the 9 $\overline{D}\to\rho\rho$ amplitudes), if one weak phase measurement is made then all of the weak phase differences are known. In principle, there are six possible weak phase differences (2 modes $\times$ 3 polarizations) which can be measured in D^0 decay to $\rho^0\rho^0$ or $\rho^+\rho^-$ so there are multiple checks on this kind of measurement.

For the $\rho\rho$ final state then we have 3 type (1) tests of the SM by comparing the magnitude of each of the $\rho^+\rho^0$ amplitudes with their conjugates. There are two other independent tests which can be made by comparing the phase differences between the amplitudes with the phase differences of their conjugates. Finally, with an absolute weak phase determination and the isospin relationships Eqn. 32 we can have 3 type (2) tests for the weak phase difference between each of the $\rho^+\rho^0$ polarizations and their conjugates.

In the case of $D\to\rho\pi$ the phase between the three $D^0\to\rho\pi$ amplitudes can be determined from analysis for the Dalitz plot distributions of $D^0\to\pi^+\pi^-\pi^0$. These amplitudes are therefore a part of a more general isospin analysis of $D\to 3\pi$ as considered in [51]. Using the relationship eqn. 35 we can see that $A^{\rho\pi}_{\frac{3}{2},2}$ is determined as a linear combination of these three amplitudes and so is determined up to an overall weak phase. Likewise we can extract the charge conjugate so the SM can be tested by comparison of $|A^{\rho\pi}_{\frac{3}{2},2}|$ with $|\overline{A}^{\rho\pi}_{\frac{3}{2},2}|$.

Furthermore, the relation eqn. 35 gives $A^{\rho\pi}_{\frac{3}{2},2}$ as a linear combination of the related charged D-meson decays $A^{\rho\pi}_{+0}$ and $A^{\rho\pi}_{0+}$ so that the phase of these two decays relative to the neutral decays can be determined up to a two fold ambiguity. We can thus use eqn. 36 to find the magnitude of $|A^{\rho\pi}_{\frac{3}{2},1}|$. Likewise we can extract the charge conjugate of the same amplitude and so SM can be tested by comparison of $|A^{\rho\pi}_{\frac{3}{2},1}|$ with $|\overline{A}^{\rho\pi}_{\frac{3}{2},1}|$. Thus we have two tests of type (1) in this system.

We can generate the corresponding type (2) tests for both of the $\Delta I=3/2$ amplitudes in $D\to\rho\pi$, if we know the weak phase difference between at least one of the neutral modes and its conjugate using the methods of Section IV. Since the relative phases between all the D decays are determined by the construction above, the weak phase difference will then be determined. The weak phases of the other two neutral cases (all of which are found in the same Dalitz plot) would then provide consistency checks.

In the case of $D_s\to K^*\pi$ there is just a SM check of type (1). In this case, the phase between the two amplitudes $A^{\pi K^*}_{+0}$ and $A^{\pi K^*}_{0+}$ can be determined from the $K^+\pi^-\pi^0$ Dalitz plot. Thus using eqn. 37 we obtain the magnitude of $|A^{\pi K^*}_{\frac{3}{2}}|$. Again we can test the SM through verifying $|A^{\pi K^*}_{\frac{3}{2}}|=|\overline{A}^{\pi K^*}_{\frac{3}{2}}|$. Unlike the above cases, there is no way to

determine the weak phase of this amplitude because there is no neutral decay related by isospin.

VI. GENERAL REQUIREMENTS FOR TESTING CP VIOLATION IN SCS DECAYS OF D-MESONS

In order to form a rough estimate of the requirements to find CP violation and test the SM through the modes above let us assume that the CP violation in these SCS modes is generally at the same level as seen in the SCS modes (e.g. $\pi\pi$ and KK), on the order of $0.1-1\%$. In terms of raw statistics, a sample of 10^5-10^7 would be required. Since the branching ratios of these modes is typically 10^{-3} this would mean that 10^8-10^{10} D-mesons would be required; and probably an order of magnitude more depending on the acceptance for various decay modes. Indeed this is roughly true in the LHCb results [1] based on an integrated luminosity of $0.62fb^{-1}$ the yield of K^+K^- was 1.44×10^6 and the yield of $\pi^+\pi^-$ was 0.38×10^6. These results point out important challenges which must be overcome to carry out such studies at LHCb and more generally at other facillities.

At the LHCb it is, of course crucial to overcome the fact that the initial state is not charge conjugate. This, of course, is less of a problem at e^+e^- colliders such as B-factories or tau-charm factories. In any case, aside from the requirement of raw statistics, it is necessary to identify and tag the initial D-meson and find the various final states. To this end, as discussed in Section III, final states with all charged final state particles (i.e. $\pi^\pm$ and $K^\pm$) will be easier to detect.

Determining the phase through any of the methods discussed in section IV may require statistics somewhat beyond currently planned facilities. First consider using straight D oscillation with Eqn. 20. Obviously the first requirement is the ability to track the time dependence of the decay to a precision $<< 1/\Gamma_{D^0}$. The relevant terms in the time dependence which we need to extract is the term $\propto \tau$. This term, of course, is multiplied by the relative rate of mixing $|z|\sim10^{-2}$. Furthermore, if the weak phase is similar to the observed level of CP violation in magnitude for $\pi^+\pi^-/K^+K^-$ then we would expect $arg(A\overline{A}^*)\sim1\%$. If this is indeed the case you would need 10^9 final states in order to see the decay and, since the branching ratio is 10^{-3} you would therefore need $\sim10^{12}$ D-mesons to start with.

Using the double oscillation method, for example eqn. 24 in the B_d case where y is small and $p/q=e^{2i\beta}$, the relevant term would be the one proportional to S_x^B. If there was no weak phase then this would be proportional to the same $\sin 2\beta$ as $B\to\psi K_s$. In effect then, we would be looking for a deviation from the SM value of this coefficient by $O(1\%)$ so we would expect to need $\sim10^5$ decays to preform the measurement. In the case of the $\pi\pi$ final state, the combined branching ratio would be 4.2×10^{-7} for the channel through either $D^0\pi^0$ or $D^0\rho^0$. This gives an initial requirement for the number of B-mesons to be $\sim 2\times10^{11}$. By combining a number of modes (e.g. $B^0\to\overline{D}^0\pi$, $B^0\to\overline{D}^0\rho$, $B^0\to\overline{D}^{0*}\pi$ etc.) it may be possible to reduce this to the $10^{10}-10^{11}$ range.

Using the correlation method, if we take the decay $B^+\to K^+D^0\overline{D}^0$ and use the index decay $g=K_s\pi^0$ with the final state $f=\pi^+\pi^-$ and assuming we need to observe 10^5 events, then, not including acceptance, the numbers of B mesons needed is 3×10^{12}. If we broaden the method to include $B^+\to K^+D^{*0}\overline{D}^{*0}$ (assuming a total Br=1%) and use as an index state $g=K_s\pi^+\pi^-$ and a target state $f=\rho^0\rho^0$, the number is reduced to 1.5×10^{11}.

Using correlations at a ψ'' factor, the number of DD pairs required using the above assumptions with index state $g=K_s\pi^+\pi^-$ and a target state $f=\rho^0\rho^0$ is 5×10^9.

It seems then that each of these methods requires an input of $\sim10^{11}$ mesons if the phase is of the same order of magnitude as A_{CP} for $\pi\pi$ and KK. This is probably beyond the capability of machines in the foreseeable future. If, however, the CP violating phase is an order of magnitude larger than A_{CP} (i.e. because the strong phase was O(10%)) then these requirements would be reduced by 2 orders of magnitude and perhaps such experiments could become possible at super B factories or the LHCb upgrades.

Perhaps the cleanest environment to measure such phases would be at high luminosity charm factories where $\sim10^{10}$ meson pairs would be needed if the phase is O(1%). Again if the phase were 10% this would be reduced by two orders of magnitude.

VII. SUMMARY AND CONCLUSION

D^0 mixing is unique as its the only charge 2/3 bound system providing us with a great opportunity to search for new physics. In many interesting BSM scenarios enhanced mixing and also enhanced CP asymmetries are expected; warped extra dimension models are a well known example. The recent discovery of direct CP violation in D^0-decays by the LHCb collaboration gives a huge impetus to these searches. The observed CP asymmetry of O(0.5%) is somewhat bigger than some estimates though it seems SM explanation is quite plausible. Hadronic uncertainties make precise predictions exceedingly difficult, therefore, for now, possible role of new physics cannot be ruled out.

More experimental information may well be pivotal in this instance This is the basic rationale behind this work leading us to make several suggestions.

We suggest that the observed enhanced effects due to non-perturbative physics may be most pronounced for the exclusive two pseudoscalar modes only, *e.g.* $\pi\pi$ and $K\overline{K}$. For multipaticle (inclusive) final states, the quark level CP asymmetry of about 6 $\times 10^{-4}$ may be releava nt. A simple way to implement this experimentally may be to look for (say), decays of D to final states with a K and a $\overline{K}$ where the sum of their energies is less than the energy of the parent D. If these inclusive final states also show enhanced CP asymmetries (say at the level seen in exclusive K^+K^-, $\pi^+\pi^-$), then that would mean that it has a new physics origin otherwise it will give support to a SM explanation.

Since the tree contribution is likely suppressed in color-suppressed final states, it is likely that CP asymmetries will be enhanced therein. To fecilitate experimental detection final states leading to charged πs may be best to focus on. These twin considerations lead us to suggest $D^0 \to \rho^0\rho^0$, $D_s \to \rho K^+$ and $D_s \to \rho^0 K^{*+}$ especially interesting. The vector vector final states have the additional bonus that angular correlations can also be used for additional CP-violating observables.

The importance of CPT constraints on CP violating observables are emphasized and illustrated with regard to exlusive and inclusive and radiative modes.

While SU(3) and Uspin symmetries seem quite badly broken in D decays, isospin likely holds quite well motivating us to to investigate its use especially in decays such as $D \to \pi\pi$, $\rho\pi$, $\rho\rho$ as well as for $D_s \to K^*\pi$.

We also studied how such analysis may be augmented by information about the weak phase in D^0 decays. To do this it is necessary to study a sample of D-mesons which are in a mixed state of D^0 and $\overline{D}^0$. Such a state may result from $D^0\overline{D}^0$ oscillation or from the decay of a B or B_s meson which itself is in a mixed state due to its oscillation. Alternatively, if a $D^0\overline{D}^0$ pair is in an entangled state, the observation of the decay of one neutral D-meson implies the other D-meson is in a mixed state. Such entangled pairs may be produced in charm factories through the decay of $\psi^{''}$ or as the result of B-meson decays.

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